2024 Online Physics Olympiad: Open Contest Solutions

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Instructions

If you wish to request a clarification, please use this form. To see all clarifications, see this document.

- Use $g = 9.8$ m/s^2 in this contest, **unless otherwise specified**. See the constants sheet on the following page for other constants.
- This test contains 35 short answer questions. Each problem will have three possible attempts.
- The weight of each question depends on our scoring system found here. Put simply, the later questions are worth more, and the overall amount of points from a certain question decreases with the number of attempts that you take to solve a problem as well as the number of teams who solve it.
- Any team member is able to submit an attempt. Choosing to split up the work or doing each problem together is up to you. Note that after you have submitted an attempt, your teammates must refresh their page before they are able to see it.
- Answers should contain **three** significant figures, unless otherwise specified. All answers within the 1% range will be accepted.
- When submitting a response using scientific notation, please use exponential form. In other words, if your answer to a problem is $A \times 10^B$, please type AeB into the submission portal.
- A standard scientific or graphing handheld calculator *may* be used. Technology and computer algebra systems like Wolfram Alpha or the one in the TI Nspire will not be needed, but they may be used.
- You are *allowed* to use Wikipedia or books in this exam. Asking for help on online forums or your teachers will be considered cheating and may result in a possible ban from future competitions.
- Top scorers from this contest will qualify to compete in the Online Physics Olympiad *Invitational Contest*, which is an olympiad-style exam. More information will be provided to invitational qualifiers after the end of the *Open Contest*.
- In general, answer in SI units (meter, second, kilogram, watt, etc.) unless otherwise specified. Please input all angles in degrees unless otherwise specified.
- If the question asks to give your answer as a percent and your answer comes out to be "x%", please input the value x into the submission form.
- Do not put units in your answer on the submission portal! If your answer is "x meters", input only the value x into the submission portal.
- **Do not communicate information to anyone else apart from your team-members before August 25, 2024**.

List of Constants

- Proton mass, $m_p = 1.67 \cdot 10^{-27}$ kg
- Neutron mass, $m_n = 1.67 \cdot 10^{-27}$ kg
- Electron mass, $m_e = 9.11 \cdot 10^{-31}$ kg
- Avogadro's constant, $N_0 = 6.02 \cdot 10^{23}$ mol^{-1}
- Universal gas constant, $R = 8.31$ J/(mol $\cdot$ K)
- Boltzmann's constant, $k_B = 1.38 \cdot 10^{-23}$ J/K
- Electron charge magnitude, $e = 1.60 \cdot 10^{-19}$ C
- 1 electron volt, 1 eV $= 1.60 \cdot 10^{-19}$ J
- Speed of light, $c = 3.00 \cdot 10^{8}$ m/s
- Universal Gravitational constant,
$$G = 6.67 \cdot 10^{-11}\ (\mathrm{N \cdot m^2})/\mathrm{kg}^2$$
- Solar Mass
$$M_\odot = 1.988 \cdot 10^{30}\ \mathrm{kg}$$
- Acceleration due to gravity, $g = 9.8$ m/s^2
- 1 unified atomic mass unit,
$$1\ \mathrm{u} = 1.66 \cdot 10^{-27}\ \mathrm{kg} = 931\ \mathrm{MeV/c^2}$$
- Planck's constant,
$$h = 6.63 \cdot 10^{-34}\ \mathrm{J \cdot s} = 4.41 \cdot 10^{-15}\ \mathrm{eV \cdot s}$$
- Permittivity of free space,
$$\epsilon_0 = 8.85 \cdot 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$$
- Coulomb's law constant,
$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \cdot 10^{9}\ (\mathrm{N \cdot m^2})/\mathrm{C^2}$$
- Permeability of free space,
$$\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{T \cdot m/A}$$
- Magnetic constant,
$$\frac{\mu_0}{4\pi} = 1 \cdot 10^{-7}\ (\mathrm{T \cdot m})/\mathrm{A}$$
- 1 atmospheric pressure,
$$1\ \mathrm{atm} = 1.01 \cdot 10^{5}\ \mathrm{N/m^2} = 1.01 \cdot 10^{5}\ \mathrm{Pa}$$
- Wien's displacement constant, $b = 2.9 \cdot 10^{-3}$ m $\cdot$ K
- Stefan-Boltzmann constant,
$$\sigma = 5.67 \cdot 10^{-8}\ \mathrm{W/m^2/K^4}$$

Problems

1. Jaywalking You are a jaywalker at a distance $x = 8.00$ m from an intersection. At time $t = 0$ you start to cross it at a constant velocity perpendicular to the direction of the street such that it would take you time T to cross it. Incidentally, at $t = 0$, there is also a car at $X = 40.0$ m away from the intersection on the same side as you. It is a self-driving Tesla, and it drives toward the intersection at such a speed that if the light remained green, the car would reach the intersection at $t = T$.

At time $t = 0$, the light turns red, and since the car is driven by a computer, it immediately begins decelerating with constant acceleration a such that it comes to a stop at exactly $X = 0$.

Do you get hit by the car? If yes, report the speed of the car relative to the ground when it hits you, as a percent of its original speed before the light turned red. If no, report how far away the car is from you when you finish crossing the street.

Solution 1: The car's initial velocity is $v_0 = X/T$. Using $v^2 = 2a\Delta x$, we find its acceleration is $a = \frac{X}{2T^2}$. Therefore, the distance the car travels in time T is

$$v_0 T - \frac{1}{2}aT^2 = \frac{3}{4}X = 30.0\ m$$

You do not get hit by the car, and you are $\boxed{2.0 \text{ m}}$ away from the car when you finish crossing the street.

2. Motorized Pendulum 1 A pendulum is made of a massless rod of length $l = 0.5000$ m and a point mass $m = 15.00$ kg hanging at one end. The angle between the rod and the vertical is θ. A motor attached to the pivot supplies a torque. The maximum value of this torque is angle-dependent and is given by $\tau(\theta) = \frac{1+\cos\theta}{2}\tau_0$ for $0 \leq \theta \leq 90°$.

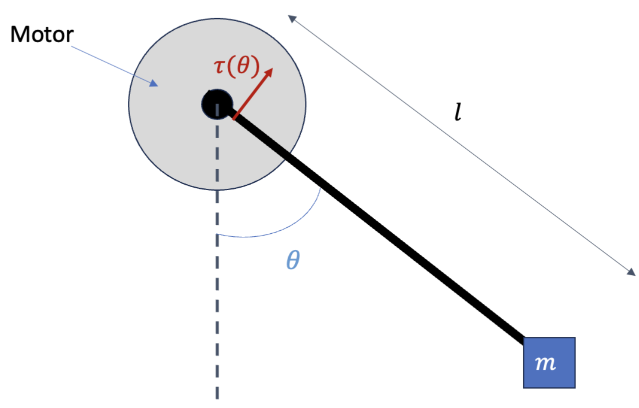


Figure 1: Motorized pendulum

The pendulum initially is given a small angular velocity counterclockwise and is at $\theta = 0$. The mass is extremely sensitive and cannot tolerate high speeds. Therefore, assume the motor always supplies just enough torque for the mass to move at a negligibly small constant speed. What is the minimum value of τ_0 needed so that the pendulum eventually reaches $\theta = 90°$?

Solution 2: The motor has the worst leverage at $\theta = 90°$. Balancing torques,

$$\frac{\tau_0}{2} = mgl$$

The answer is $\boxed{\tau_0 = 2mgl = 147.1 \text{ N} \cdot \text{m}}$.

3. Motorized Pendulum 2 The pendulum initially is at $\theta = 0$. This time, the mass is not so sensitive. The motor may supply its full torque for all θ. What is the minimum value of τ_0 needed so that the pendulum reaches $\theta = 90°$ in a single unidirectional swing?

Solution 3: Our constraint here is that

$$\int_0^t (\tau(\theta) - mgl\sin\theta)\, d\theta \geq 0$$

for all $0 \leq t \leq 90°$. In other words, the cumulative energy input minus the work done by gravity should never be negative. Carrying out the integral and simplifying a bit, we get the following inequality:

$$\frac{\tau_0}{2mgl} \geq \frac{1 - \cos t}{t + \sin t}$$

Between zero and ninety degrees, the function on the left-hand side is monotonically increasing. So, to find its maximum, evaluate it at $t = \pi/2$.

Our answer is $\boxed{\tau_0 = \frac{2}{1 + \pi/2} mgl = 57.1 \text{ N} \cdot \text{m}}$.

4. Motorized Pendulum 3 The pendulum initially is at $\theta = 0$. The mass contains extremely sensitive electronics that cannot tolerate speeds above $v_{max} = 0.1000$ m/s. To three significant figures, what is the minimum value of τ_0 needed so that the pendulum reaches $\theta = 90°$ without exceeding this speed threshold?

Solution 4: The optimal route is to accelerate the mass to v_{max}, then as it approaches $\theta = 90°$, let the mass's kinetic energy carry it to the top while the motor fails to provide enough torque to counteract gravity.

We will approximate the answer. Our setup is similar to Motorized Pendulum 1, so the answer to this one should be a small deviation from the answer to that question. Namely, let $\tau_0 = 2mgl - \epsilon$. Near ninety degrees, the torque due to gravity is roughly mgl, and the motor's output torque is roughly $\tau_0 \frac{1+\delta}{2}$, where $\delta = 90° - \theta$ is the distance to the top.

We can find the point at which gravity begins to overpower the motor by setting mgl equal to $\tau_0 \frac{1+\delta}{2}$. This happens at

$$\delta = \epsilon/\tau_0 \approx \epsilon/2mgl$$

Now, from this point onward, the work done by gravity minus the work done by the motor will be positive, causing the mass to slow down. However, this net work should never be larger than the kinetic energy of the mass, $\frac{1}{2}mv^2$. Approximating the torques from gravity and the motor as lines, the net torque is also a line, and we can therefore calculate how much work is done to slow down the mass from the point at which gravity overpowers the motor to ninety degrees. The net torque is

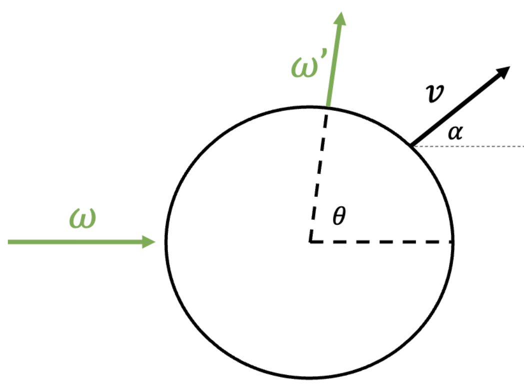
$$mgl - \tau_0 \frac{1+\delta}{2} \approx mgl - 2mgl\frac{1+\delta}{2} = -mgl\delta$$

The area under the graph (i.e. net work) from $\delta = \epsilon/2mgl$ to zero is

$$\frac{1}{2} \cdot \epsilon/2mgl \cdot \epsilon/2 = \frac{\epsilon^2}{8mgl}$$

We can solve for ϵ by setting this area equal to $\frac{1}{2}mv^2$, resulting in $\epsilon \approx 6.6\ Nm$. Therefore, the answer to three significant figures is $\boxed{\tau_0 = 147.1 - 6.6 = 140.\ \mathrm{N \cdot m}}$.

5. Relativistic Scattering A small spherical particle traveling at a speed $v = 0.5c$ at an angle $\alpha = 45°$ from the horizontal is struck by an electromagnetic plane wave of angular frequency $\omega = 7.08 \times 10^{15}$ Hz propagating directly to the right. In its own reference frame, the particle scatters light in all directions with the same frequency as the frequency of incident light it perceives. Due to the relativistic Doppler effect, however, the frequency of the scattered light measured in the lab frame is generally not the same as the incident light frequency. What is the angular frequency ω' of light scattered into a scattering angle of $\theta = 89°$? Assume the radius of the particle R is small enough that $R\omega \ll c$.



Solution 5:
We first transform into the reference frame of the particle. In this frame, it becomes a stationary particle receiving and emitting light isotropically. What frequency of light, we ask? We will perform a Lorentz transformation to find out. Let the x-axis be along the direction of the particle velocity. Setting $c = 1$, the wavevector 4-vector in the lab frame is

$$k^\mu = (\omega, \omega\cos\alpha, -\omega\sin\alpha)$$

Transforming to the particle frame, the new 4-vector is

$$k_1^\mu = (\omega\gamma(1 - v\cos\alpha), ..., ...)$$

The x- and y- components do not matter as we only care about the t-component, which tells us the frequency of the light in the particle frame. So light of frequency $\omega_1 = \omega\gamma(1 - v\cos\alpha)$ gets scattered into all directions. Consider light that gets scattered into an arbitrary angle ϕ from the x-axis. Its 4-vector would be

$$k_2^\mu = (\omega_1, \omega_1\cos\phi, \omega_1\sin\phi)$$

Transforming this back into the lab frame, the final 4-vector is

$$k_3^\mu = (\omega_1\gamma(1 + v\cos\phi), \omega_1\gamma(\cos\phi + v), \omega_1 \sin\phi)$$

In the lab frame, the angle δ between the x-axis and the direction of propagation of k_3 is given by

$$\tan\delta = \frac{\sin\phi}{\gamma(\cos\phi + v)}$$

We can start plugging in numbers. We find that $\gamma = 1.155$, $\omega_1 = 5.29 \cdot 10^{15}$ Hz, and $\tan\delta = 0.966$. We can solve for ϕ, getting back $\phi = 70.0°$.
Our answer is therefore $\boxed{\omega' = k_3^0 = 7.15 \cdot 10^{15} \text{ Hz}}$.